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Mathematics

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Is There Evidence of God From the Philosophy of Mathematics??

The foregoing is only the first step in the philosophical proof of a Creator of past time. It shows that there is an analytical contradiction in the concept of "infinite past time" which strongly implies that this concept cannot exist in reality (and that past time must therefore be finite). If readers are interested in the full a-priori synthetic proof for a beginning of past time (which necessitates a Creator of past time which itself is not in past time) then they may want to consult Chapter Five of NPEF and units 22 through 26 of PMD (see the Introduction to this book – Footnote #1).

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An Analytical Contradiction in "Infinite Past Time"

The problematic character of infinite past time is revealed by a seemingly inescapable analytical contradiction in the very expression "infinite past time."

If one splits the expression into its two component parts: (1) "past time" and (2) "infinite," and attempts to find a common conceptual base which can apply to both terms (much like a lowest common denominator can apply to two different denominators in two fractions), one can immediately detect contradictory features. One such common conceptual base is the idea of "occurrence," another, the idea of "achievement," and still another, the idea of "actualizability." Let us begin with the expression "past time."

Past time can only be viewed as having occurred, or having been achieved, or having been actualized; otherwise, it would be analytically indistinguishable from present time and future time. In order to maintain the analytical distinction among these three interrelated ideas, present time must be viewed as "occurring," or "being achieved," or "being actualized"; and future time must be viewed as "not having occurred," "not achieved," and "not actualized." The notion of "past" loses its intelligibility with respect to present and future if its meaning were to include "occurring," "being achieved," or "being actualized" (pertaining to the present); or "not having occurred," "not achieved," or "not actualized" (pertaining to the future). If past time is to retain its distinct intelligibility, it can only be viewed as "having occurred," "achieved," and "actualized."

Now let us turn to the other side of the expression, namely, "infinite." Throughout this Unit, I will view "infinity" within the context of a continuous succession because I will show that real time in changeable universes must be a "continuous succession of non-contemporaneous distension." Now, infinities within a continuous succession imply "unoccurrable," "unachievable," and "unactualizable," for a continuous succession occurs one step at a time (that is, one step after another), and can therefore only be increased a finite amount. No matter how fast and how long the succession occurs, the "one step at a time" or "one step after another" character of the succession necessitates that only a finite amount is occurrable, achievable, or actualizable. Now, if "infinity" is applied to a continuous succession, and it is to be kept analytically distinct from (indeed, contrary to) "finitude," then "infinity" must always be more than can ever occur, be achieved, or be actualized through a continuous succession ("one step at a time" succession). Therefore, infinity would have to be unoccurrable, unachievable, and unactualizable when applied to a continuous succession. Any other definition would make "infinity" analytically indistinguishable from "finitude" in its application to a continuous succession. Therefore, in order to maintain the analytical distinction between "finitude" and "infinity" in a continuous succession, "infinity" must be considered unoccurrable (as distinct from finitude which is occurrable), unachievable (as distinct from finitude which is achievable), and unactualizable (as distinct from finitude which is actualizable). We are now ready to combine the two parts of our expression through our three common conceptual bases:

"Infinite.............................Past Time"

"(The) unoccurable................(has) occurred."

"(The) unachievable...............(has been) achieved."

"(The) unactualizable.............(has been) actualized."

Failures of human imagination may deceive one into thinking that the above analytical contradictions can be overcome, but further scrutiny reveals their inescapability. For example, it might be easier to detect the unachievability of an infinite series when one views an infinite succession as having a beginning point without an ending point, for if a series has no end, then, a priori, it can never be achieved. However, when one looks at the infinite series as having an ending point but no beginning point (as with infinite past time reaching the present), one is tempted to think that the presence of the ending point must signify achievement, and, therefore, the infinite series was achieved. This conjecture does not avoid the contradiction of “infinite past time” being “an achieved unachievable.” It simply manifests a failure of our imagination. Since we conjecture that the ending point has been reached, we think that an infinite number of steps has really been traversed, but this does not help, because we are still contending that unachievability has been achieved, and are therefore still asserting an analytical contradiction.

Another failure of our imagination arises out of thinking about relative progress in an historical succession. Our common sense might say that infinite past history is impossible because an infinity is innumerable, immeasurable, and unquantifiable, making the expression “an infinite number” an oxymoron. But then we get to thinking that infinite history seems plausible because each step relative to the other steps is quantifiable in its progression; each step is subject to relative numeration. Therefore, it seems like history can really achieve an infinite number of steps.

However, as the above analysis reveals, this cannot be so because an infinity in a continuous succession must be unachievable or unactualizable as a whole (otherwise, it would be analytically indistinguishable from “finitude” in a continuous succession). Since, as has been said, past time must be achieved or actualized (otherwise it would be analytically indistinguishable from “present” and “future”), “infinite past time” must be an “achieved unachievable” or an “actualized unactualizable” (an intrinsic contradiction). Moreover, the expression “an infinite number” is also an intrinsic contradiction because “number” implies a definite quantity, whereas “infinity” implies innumerability (more than can ever be numbered). Therefore, infinite history and its characterization as “a completion of infinite time,” remains inescapably analytically contradictory.

This intrinsic analytical contradiction reveals the problematic character of the very idea of “infinite past time.” It now remains for us to show the inapplicability of this problematic idea to our universe, and indeed, to any really possible changeable universe. This step will give ontological (“synthetic”) significance to the analytical contradiction by showing that the condition of the real world (i.e., our real universe, or any really possible changeable universe) will contradict (and therefore resist) the application of this problematic idea to it. The result will be that no real universe could have had infinite past time.

Before we can proceed to this proof, we must first give an ontological explanation of real time[1], and then show that this real time must be intrinsic to any changeable universe, and then explain Hilbert’s distinction between actual and potential infinities so that it will be clear that “infinite past time” (as defined) must be an actual infinity which Hilbert shows to be inapplicable to any reality to which the axioms of finite mathematics can be applied. The ontological proof against an infinity of past time will follow from this.

Please note that the a-priori synthetic proof is based upon the work of the famous mathematician David Hilbert (see “On the Infinite” in Hilary Putnam, ed Philosophy of Mathematics (Inglewood Cliffs, NJ: Prentiss Hall, 1977). What follows is a brief summary of Hilbert’s discovery. For a full explanation see NPEG, Chapter V, and PID Units 22 – 27.

A Brief Explanation of Real Time

It is perhaps best to begin our ontological analysis of time without making recourse to locomotion (which combines space and time). This may be done by looking at a non-spatial change such as death. Let us suppose a cat dies. One of the most apparent ontological truths about this occurrence is that “the state before” and “the state after” cannot be coincident. If they were, it would be an obvious contradiction (the cat simultaneously alive and dead). This, of course, is the problem with all history. Changed existential states in any specific entity cannot be coincident without contradiction. Therefore, wherever there is change, indeed, wherever there is changeability, there must also be some existential non-coincidence which allows differing states to occur within a single entity (e.g., a cat). Let us sum up this initial definition of time as “the existential non-coincidence necessary for the possibility of changed states within a single entity.” If this existential non-coincidence were not objectively real, changeable beings and changeable states within the same being would have to be simultaneous, and therefore intrinsically contradictory, and therefore impossible. In view of this, time may also be defined as, “that without which all history is a contradiction.”

At this point, one will want to ask, “What is ‘existential non-coincidence?’” or “How does it manifest itself?” The temptation here is to spatialize it, by, for example, inserting a spatial continuum between “the cat alive” and “the cat dead.” Though this may be very satisfying from the vantage point of human imagination, it leads to a host of problems. To begin with, our cat both alive and dead is in the same place, and the separation of its existential states is not describable by an extensive – spatial – separation. Yet, the cat’s change does require a non-extensive separation (frequently termed “a distensive separation”). One must be careful here not to visualize distensive separation as a three-dimensional continuum, otherwise one will be imposing a quasi-spatial continuum between events.

Henri Bergson wrestled with this problem, and finally made recourse to a kind of “protomentalist unified separation of existential states” which he termed “elementary memory.” He supposed that this elementary memory existed in the universe as a whole, as a kind of very “elementary cosmic consciousness.” In a famous passage in Duration and Simultaneity, he noted:

What we wish to establish is that we cannot speak of a reality which endures without inserting consciousness into it.[2]

In order to show this, he constructs a thought experiment in which he assumes the above existential non-coincidence of incompatible states:

We shall have to consider a moment in the unfolding of the universe, that is, a snapshot that exists independently of any consciousness, then we shall try conjointly to summon another moment brought as close as possible to the first, and thus have a minimum amount of time enter into the world without allowing the faintest glimmer of memory to go with it. We shall see that this is impossible. Without an elementary memory that connects the two moments, there will be only one or the other, consequently a single instant, no before and after, no succession, no time.[3]

I do not wish here to either affirm or deny Bergson's protomentalist conclusions, but I do want to acknowledge the ontological conditions of change and time which Bergson recognized in concluding to them, namely,

1) a real existential non-coincidence between changed states,

2) a fundamental unity within this separation which unifies the non-coincidence of earlier and later, and

3) the non-spatial (and hence, for Bergson, the "elementary memory" or "elementary consciousness") character of this "unity of existential non-coincidence."

These three ontological conditions now give a further refinement of our ontological explanation of time, namely, "a non-spatial unity intrinsic to existential non-coincidence necessary for changeability." Inasmuch as this unity is divisible into "earlier" and "later" (as Bergson correctly surmises) it is a non-contemporaneous manifold. This non-contemporaneous manifold is distinct from a spatial unity which is a contemporaneous manifold. Since the transition from earlier to later is akin to a "stretching from within," I will refer to it as "distension" instead of "extension" which more properly applies to a contemporaneous (spatial) manifold. Hence, "real time" may now be defined as a "non-contemporaneous" distensive manifold intrinsic to changeable realities (or groups of changeable realities)."

Summary of Hilbert's Prohibition of Actual Infinities

In order to expedite the explanation of Hilbert's prohibition, it will be helpful to draw a distinction between three kinds of infinity which are genuinely distinct from one another and cannot be used as analogies for one another. This will show why Hilbert's prohibition only applies to C infinities (infinities hypothesized to be within algorithmically finite structures).

Three Kinds of Infinity

For the sake of convenience, I will term these three kinds of infinity A, B, and C:

1) "A-infinity." "Infinite" frequently has the meaning of "unrestricted," (e.g., "infinite power" means "unrestricted power"). It can only be conceived through the "via negativa," that is, by disallowing or negating any magnitude, characteristic, quality, way of acting, or way of being

which could be restricted or introduce restriction into an infinite (unrestricted) power. Therefore, "infinite," here, is not a mathematical concept. It is the negation of any restriction (or any condition which could introduce restriction) into power, act, or being.

2) "B-infinity." "Infinite" is also used to signify indefinite progression or indefinite ongoingness. An indefinite progression is never truly actualized. It is one that can (potentially) progress ad infinitum. Examples of this might be an interminably ongoing series, or an ever-expanding Euclidean plane. The series or the plane never reaches infinity; it simply can (potentially) keep on going ad infinitum. Thus, Hilbert calls this kind of infinity a "potential infinity."

3) "C-infinity." "Infinite" is sometimes used to signify "infinity actualized within an algorithmically finite structure." Mathematicians such as Georg Cantor hypothesized a set with an actual infinite number of members (a Cantorian set) which would not be a set with an ever-increasing number of members or an algorithm which could generate a potential infinity of members. Examples might be an existing infinite number line, or an existing infinite spatial manifold, or the achievement of an infinite continuous succession of asymmetrical events (i.e., infinite past history).

The Hilbertian prohibition applies to the C-infinity alone, for, as will be seen, it is not concerned with non-mathematical infinities (i.e., an A-infinity), and it permits indefinitely ongoing (continually potential) successions through algorithmically finite structures (i.e., B-infinities). Before showing Hilbert's and others' prohibition of C-infinities, the two permissible kinds of infinities will be discussed.

An A-infinity has long been recognized by the Scholastic tradition.[4] As noted above, it is not a mathematical infinity (such as infinite sets, infinite number lines, infinite successions, etc.) and it is not applied to algorithmically finite structures (such as spatial magnitudes, temporal magnitudes, fields, forces, etc.). Hence, it does not postulate an infinite Euclidean plane, infinite past time, an infinite number line, infinite space, infinite history, infinite thermometers, infinite density, or an infinite physical force. An A-infinity is simply the recognition of "non-restrictedness" in power. It is, therefore, a negation of any predicate which has restriction or could imply restriction in an infinite power.

As Scholastic philosophers have long recognized, one can only speak about "infinite power" or "infinite being" by negating any restriction (or structure giving rise to a restriction such as a divisible magnitude) to the power itself. Thus, one can say that "infinite power" is not restricted as to form, way of acting, space-time point, or even to spatiality itself (which is a divisible magnitude having intrinsically finite parts).

Such negative statements are not equivalent to "no knowledge" or unintelligibility; for one does know that infinite power does not have a restriction. Yet, at the same time, one cannot positively imagine (through, say, picture-thinking) what such unrestricted power would be. Every image we have is likely to restrict the entity we are conceiving either intrinsically or extrinsically.

Our inability to conceive or imagine this entity does not in any way rule out its possibility, for our inability to conceive of it does not reveal an intrinsic contradiction or "an extrinsic contradiction with some existing reality;" it merely admits the limits of our spatializing, temporalizing, finitizing imagination and conception. Thus, as we shall see, Hilbert's prohibition of an "actual infinity" does not extend to an A-infinity, for an A-infinity is neither a mathematical infinity nor an application of infinity to an algorithmically finite structure. Interestingly enough, Hilbert's prohibition of a C-infinity could actually constitute a proof for an A-infinity.

A B-infinity is quite distinct from an A-infinity because it is both a mathematical infinity and an application of infinity to an algorithmically finite structure. Unlike the prohibited C-infinity, the B-infinity applies a mathematical infinity to an algorithmically finite structure in only a potential way. Therefore, the B-infinity only acknowledges the possibility that an algorithmically finite structure could continue to progress indefinitely.

Thus, the B-infinity does not imply that a Cantorian set (with an infinite number of members) actually exists. It implies that a particular algorithm (sufficient to define the set) can continue to generate members indefinitely. Furthermore, it does not hold that an infinite number line actually exists, but rather than one can continue to generate numbers on the line indefinitely. The existence (completion or achievement) of an infinite number line is never advocated, but only the potential to continue to generate numbers according to a particular algorithm indefinitely.

The same holds true for magnitudes such as space (a contemporaneous magnitude) and time (a non-contemporaneous magnitude). A potential infinity implies that a spatial magnitude has the potential to continue expanding indefinitely. Similarly, it holds that a non-contemporaneous succession of events has the potential to continue indefinitely (into the future). It does not imply that an infinite spatial magnitude really exists or that an infinity of continuously successive historical events actually occurred.

The Hilbertian prohibition does not apply to a B-infinity because one is not advocating the existence (actuality) of a mathematical infinity within an algorithmically finite structure. One is only advocating the potential to increase an algorithmically finite structure indefinitely according to a particular algorithm. As we shall see momentarily, infinity applied to the succession of future events will not give rise to a Hilbertian paradox because future events are only potential. An infinity never exists. Future time can only be an indefinitely increasing succession of events; never the existence (actuality) of a mathematical infinity. As will be seen, such is not the case with past time, which explains why infinite past time falls under the Hilbertian prohibition.

A C-infinity, like a B-infinity, is both a mathematical infinity and an application of infinity to an algorithmically finite structure. The important difference, however, between the B and C-infinities is that the C-infinity implies the existence (actuality) of a mathematical infinity within an algorithmically finite structure. As noted above, examples of a C-infinity would be an actual

Cantorian set with an actual infinite number of members, or an infinite number line with an actual infinite number of positions, or an actually existing infinite spatial magnitude, or an actual occurrence of an infinite number of events in the past. Thus, if C-infinities could really exist, there could be infinite space, infinity degrees Fahrenheit, infinite mass density, infinite physical force, and infinite past time. These notions seem irresolvably paradoxical prima facie, because the mathematical infinity applied to them utterly destroys their intelligibility as algorithmically finite structures. The proof for this goes beyond prima facie intuition. It extends to the requirements for mathematical intelligibility itself. Thus, as Hilbert shows, a mathematical infinity existing within an algorithmically finite structure undermines the very possibility of finite mathematics, and therefore the very possibility of quantifying those algorithmically finite structures. Therefore, a C-infinity must, in all cases, be illusory.

Now, it was shown above that the succession of past events is a real, non-contemporaneously distended, interactive, asymmetrically related, continuously successive whole. As such, it must be an actual asymmetrical progression. It does not matter that past events no longer exist, because all past events did exist and affected, and were related to, present events as they passed out of existence. Thus, they constitute a real past progression. This is sufficient to qualify "a past succession of events" for Hilbert's prohibition, because the application of an infinity to it must be a C-infinity (not a B-infinity).

If a C-infinity must in all cases be rejected (because it entails the undermining of finite mathematics and the quantification of algorithmically finite structures), then an infinite past succession of events must also be rejected. This will be shown first by summarizing Hilbert's (and others') prohibition of C-infinities and second through a formal proof which illustrates the contradictory and incoherent nature of the C-infinity applied to past time.

It is important not only to distinguish among these three kinds of infinity, but also to avoid analogizing one with the other. Thus, infinite future time cannot be a proper analogy for infinite past time. As can be seen, they are quite distinct (a B-infinity versus a C-infinity, respectively). Furthermore, infinite future time cannot be used as an analogy for infinite power (a B-infinity versus an A-infinity, respectively). The rules for each kind of infinity do not apply meaningfully to the other kinds.

The Mathematical Prohibition of C-Infinities

The above discussion was brought to the attention of philosophers of mathematics by David Hilbert, who attempted to clarify the notion of an infinite numeric series which was thought to exist as a completed totality:

Just as in the limit processes of the infinitesimal calculus, the infinite in the sense of the infinitely large and the infinitely small proved to be merely a figure of speech, so too we must realize that the infinite in the sense of an infinite totality, where we still find it used in deductive methods, is an *illusion*.[5]

Hilbert is proposing here that, even though a B-infinity (one with the potential to continue indefinitely without being actual) is mathematically admissible, a C-infinity (the existence of a mathematical infinity within an algorithmically finite structure) is not mathematically admissible because it presents irresolvable paradoxes and contradicts the very axioms of finite mathematics. In recounting the history of the "actual infinite" (Hilbert's designation of a C-infinity from Georg Cantor's actual infinite set of numbers) Hilbert notes that the Russel-Zermelo paradox presents so many devastating contradictions that it nearly undermined deductive procedure in mathematics:

These contradictions, the so-called paradoxes of set theory, though at first scattered, became progressively more acute and more serious. In particular, a contradiction discovered by Zermelo and Russell had a downright catastrophic effect when it became known throughout the world of mathematics. Confronted by these paradoxes, Dedekind and Frege completely abandoned their point of view [belief in the coherency of an infinite set as proposed by Cantor] and retreated.[6]

Hilbert then concludes that the technique of ideal elements (which can imply infinities) cannot be used if they change the fundamental axioms of finite numbers to which they have been applied. Since this does not occur with potential infinities (B-infinities), but always occurs with actual infinities (C-infinities), Hilbert rejects the use of the latter in any way that could apply to the real world (i.e., real magnitudes, real counting, real series, etc.):

In summary, let us return to our main theme and draw some conclusions from all our thinking about the infinite. Our principal result is that the infinite is nowhere to be found in reality. It neither exists in nature nor provides a legitimate basis for rational thought – a remarkable harmony between being and thought. … The role that remains for the infinite to play is solely that of an idea – if one means by an idea, in Kant's terminology, a concept of reason which transcends all experience and which completes the concrete as a totality [a B-infinity]…. [7]

Conclusions

Hilbert's analysis shows that the existence of a mathematical infinity in an algorithmically finite structure results not only in a contradiction, but also in the undermining of the axioms of finite numbers which it was intended to complete. If devastating consequences for the whole of mathematical reasoning are to be avoided, C-infinities must not be applied to real magnitudes, successions, series, or any algorithmically finite structure that could be considered real (such as past time).

Hilbert's prohibition of C-infinities continues to be widely held by contemporary mathematicians. As William Lane Craig notes:

According to Robinson, "Cantor's infinities are abstract and divorced from the physical world [Robinson 1969, p. 163]." This judgement is echoed by Fraenkel, who concludes that among the various branches of mathematics, set theory is "the branch which least of all is connected with external experience and most genuinely originates from free intellectual creation [Fraenkel 1973, p. 240].” As a creation of the human mind, state Rotman and Kneebone…when one selects from an infinite set an infinite subset, the actual possibility of such an operation is not implied. “The conception of an infinite sequence of choices (or of any other acts)…is a mathematical fiction – an idealization of what is imaginable only in finite cases [Rotman and Kneebone 1966, p. 60].” [8]</blockquote Of course, infinities can be applied to sets in merely theoretical or abstract ways (e.g., Cantorian sets or the Zermelo-Fraenkel universe of sets), but this cannot be thought to have applicability to the real world:

[T]he Zermelo-Fraenkel universe of sets exists only in a realm of abstract thought… [I]he “universe” of sets to which the…theory refers is in no way intended as an abstract model of an existing Universe, but serves merely as the postulated universe of discourse for a certain kind of abstract inquiry.[9]

In sum, Hilbert’s, Fraenkel’s, Rotman’s, Kneebone’s, Zermelo’s, Robinson’s, and many others’ analysis shows that the existence of a mathematical infinity in an algorithmically finite structure results not only in a contradiction, but also in the undermining of the axioms of finite numbers which it was intended to complete. If devastating consequences for the whole of mathematical reasoning (and also the applicability of mathematics to the finite universe) are to be avoided, C-infinities must not be applied to real magnitudes, successions, series, or any algorithmically finite structure that could be considered real (such as past time). At this juncture, the reader will probably notice the invalidity of the hypothesis “infinite past history” or “infinite past time.” The ontological explanation of time, which shows that history must be a continuous succession of events (each of which has real distensive separation and real power to aggregate the whole of the continuous succession) reveals that when infinity is applied to it, it must imply “infinity within an actual algorithmically finite structure,” which implies an actual infinity (a C-infinity). As noted above, this C-infinity must be considered illusory (nonexistent within a standard universe) because it undermines the axioms of finite mathematics which ground the quantifiable intelligibility of the realities in that universe (and also the applicability of mathematics to the finite universe). This deduction alone is sufficient to show that infinite history (implying infinite past time) cannot exist through any possible reality (or contemporaneously unified group of realities) in any possible universe. There will have to be a beginning (and a creator) of past time wherever past time exists. C-infinities not only undermine the axioms of finite mathematics, but also the intelligibility of the finite realities to which they have been applied. For example, the existence of an infinity in the whole of past history would undermine the distensive separation of every part of that past history (reducing its aggregative effect within the whole to nothing – a dimensionless point), because an infinite distension minus any finite part, or any infinite part which is a subset of the whole, is still infinity. But this cannot be the case in real history, because every part of past time must maintain its distensive separation and its power to aggregate the whole. If it did not, then history would be fraught with irresolvable contradictions (e.g., the cat alive and dead simultaneously).

Therefore if every part of real history (and real time) are to maintain their real distensive separation then every part of real history and time must contribute or constitute (build up) the whole of real history or time, because every part separates everything that came before it from everything coming after it. But as we saw, no part can really contribute or constitute (build up) an infinite continuum – its addition or removal has no effect – it does not change the whole at all. Inasmuch as parts in an infinite whole cannot build up the whole, they cannot cause real distensive separation of everything that came before from everything coming after in that whole and so parts of an infinite whole cannot be parts of real history or real time. If they were, history would be fraught with contradictions. This leads to the conclusion that history and time must be finite, and if finite, must have a beginning. As we saw in Unit D, a beginning of time implies a Creator. This Creator would have to be timeless.

For the demonstration of the timelessness of the Creator of pastime, see NPEG, Chap Five, Section V.

Footnotes

1. ↑ "Real time" will be defined as "a continuous succession of non-contemporaneous distention".
2. ↑ Bergson 1965, p. 48. See also Spitzer 1989, pp. 12-14.
3. ↑ Bergson 1965, p. 48.
4. ↑ Beginning with Pseudo-Dionysius the Areopagite. See Rolt 1940.
5. ↑ Hilbert 1964, p. 135.
6. ↑ Hilbert 1964, p. 141.
7. ↑ Hilbert 1964, p. 151.
8. ↑ Craig 1993, p. 10. Many other mathematicians and philosophers have recognized this as well. See, for example, Whitrow, 1954-1955, pp. 215-225; 1967, pp. 422-432; 1968; and 1970, pp. 224-233.
9. ↑ Rotman and Kneebone 1966, p. 61.